Regional Course on 2008 SNA (Special Topics): Improving Exhaustiveness of GDP Coverage

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Session 5.9
Price and Volume Measures: Chain Linked Volume Estimates

Alick Nyasulu
SIAP Statistician/Lecturer

Outline

* Why volume measures?
* Chain Indices
* Chaining in Annual National Accounts
* Chaining in Quarterly National Accounts
Analysts wish to measure the “real” growth of national accounts data

* **growth free of the direct effects of price change**

Current price values in the national accounts are subject to the effects of changing prices

* **unsatisfactory for these purposes**

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* All flows of goods and services can be decomposed into a price component $p$ and a volume component $q$. For example, consider the sale of beef and chicken:

* In period 1, 20 kilos of beef are sold at €1.00 per kilo for a value of €20.00 and 10 kilos of chicken are sold at €2.00 per kilo for a value of €20.00. Total sales of meat are valued at €40.00.

* In period 2, 18 kilos of beef are sold at €1.10 per kilo for a value of €19.80 and 12 kilos of chicken are sold at €2.00 per kilo for a value of €24.00. Total sales of meat are valued at €43.80.
Why volume measures?

* From our example: Total sales of meat have increased from €40.00 in period 1 to €43.80 in period 2. But what is the growth in “real” terms

* The total value of sales in period 2 at period 1 prices is €42.00 (18 kilos of beef @ €1.00 plus 12 kilos of chicken @ €2.00). At period 1 prices, the total value of meat sales has increased from €40.00 to €42.00, which is an increase of 5%.

Why volume measures?

This can be expressed algebraically as:

\[
\frac{p_1^1 q_1^1 + p_2^1 q_2^1}{p_1^1 q_1^1 + p_2^1 q_2^1} = \frac{(1.00 \times 18) + (2.00 \times 12)}{(1.00 \times 20) + (2.00 \times 10)} = \frac{18.00 + 24.00}{20.00 + 20.00} = \frac{42.00}{40.00} = 1.05
\]

This expression is called a Laspeyres volume index. The defining feature is that in calculating growth from one period to another, the prices of the earlier period are applied to both periods.
**Why volume measures?**

A Laspeyres volume index from year \( y-1 \) to year \( y \) is of the form:

\[
L_y^L = \frac{\sum_{i=1}^{n} P_i y Q_i y}{\sum_{i=1}^{n} P_i y-1 Q_i y-1}
\]

The denominator is the current price value of the aggregate in year \( y-1 \), and \( P_i y-1 \) and \( Q_i y-1 \) denote the prices and quantities of the \( i^{th} \) product in year \( y-1 \), and there are \( n \) products.

But this an oversimplification since NA aggregates have many products/transactions in a year.

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**Why volume measures?**

* A Paasche volume index from year \( y-1 \) to year \( y \) is defined as:

\[
P_y^y = \frac{\sum_{i=1}^{n} P_i y Q_i y}{\sum_{i=1}^{n} P_i y-1 Q_i y-1}
\]

The defining feature is that in calculating growth from one period to another, the prices of the later period are applied to both periods.
Why volume measures?

From our example, the Paasche Volume Index yields:

\[
\frac{p_{base}q_{base}}{p_{base}q_{base} + p_{base}q_{chicken}} + \frac{p_{base}q_{chicken}}{p_{base}q_{beef} + p_{base}q_{chicken}} = \frac{(1.10 \times 18) + (2.00 \times 12)}{(1.10 \times 20) + (2.00 \times 10)} = \frac{19.80 + 24.00}{22.00 + 20.00} = \frac{43.80}{42.00} = 1.043
\]

Both the Laspeyres and Paasche indices are equally valid for calculating the volume growth of meat sales between period 1 and period 2, yet they give different answers.

This suggests that an average of the two may be a better estimate than either of them. Fisher’s Ideal Index (hereafter referred to as the Fisher index) is the geometric mean of the Laspeyres and Paasche and is considered to be a superior index.

Why volume measures?

* A Fisher index is derived as the geometric mean of a Laspeyres and Paasche index:

\[
F_Q^t = \left( L_Q^t \times P_Q^t \right) \frac{1}{2}
\]
A Paasche price index from year \( y-1 \) to year \( y \) can be derived as:

\[
P_y = \frac{\sum_{i=1}^{n} P_y^i Q_i^y}{\sum_{i=1}^{n} P_{y-1}^i Q_i^{y-1}},
\]

When this Paasche price index is divided into the current price value for year \( y \) a constant price estimate is produced as below:

\[
\frac{\sum_{i=1}^{n} P_y^i Q_i^y}{P_y} \times \frac{\sum_{i=1}^{n} P_{y-1}^i Q_i^{y-1}}{\sum_{i=1}^{n} Q_i^{y-1}} = \frac{\sum_{i=1}^{n} P_y^i Q_i^y}{\sum_{i=1}^{n} P_y^i Q_i^{y-1}} = L_v^y
\]

Dividing this by the current price value of the previous year, year \( y-1 \), yields a Laspeyres volume index from year \( y-1 \) to year \( y \). Evidently, Laspeyres volume indices and Paasche price indices complement each other, and vice versa.

Note in practice, the required Paasche price indices are generally unavailable, and Laspeyres price indices are used to deflate current price values instead.
Frequent linking is beneficial when price and volume relativities change monotonically.

For example: volume estimates of gross fixed capital formation are much better derived as chain indices than as fixed-weighted indices (or constant price estimates) mainly because of the steady decline in the relative prices of computer equipment and the corresponding increase in their relative volumes.

While chain Fisher indices perform best in such circumstances, and are a much better indicator than fixed-weighted indices, chain Laspeyres indices capture much of the improvement from frequent linking.

- Frequent linking is least beneficial when price and volume relativities are volatile.
- All chained series are subject to drift when there is price and volume instability, but chain Fisher indices usually drift less than either chain Laspeyres or chain Paasche indices.
- The drift of a particular type of index is given by the difference between the index’s values at time $t$ and time $t+n$. 
In reality it is very uncommon for prices and volumes to return to the values observed in an earlier period.

The drift and long–term accuracy of a chain or fixed-weighted index can be assessed over a period of time by comparing it with a direct Fisher index, i.e. a Fisher index calculated directly from the first to the last observation in the period.

Annual chain Laspeyres and Paasche indices can be formed by multiplying consecutive year-to-year indices. Chain Fisher indices can be derived by taking their geometric mean.

\[
L_{cv}^{ch} = \frac{\sum p_i^0 q_1^0 \times \sum p_i^1 q_1^0 \times \sum p_i^2 q_1^0 \times \ldots \times \sum p_i^n q_1^n}{\sum p_i^0 q_1^0 \times \sum p_i^1 q_1^1 \times \sum p_i^2 q_1^2 \times \ldots \times \sum p_i^n q_1^n}
\] (6.7)

\[
P_{cv}^{ch} = \frac{\sum p_i^0 q_1^0 \times \sum p_i^1 q_1^1 \times \sum p_i^2 q_1^2 \times \ldots \times \sum p_i^n q_1^n}{\sum p_i^0 q_1^0 \times \sum p_i^1 q_1^1 \times \sum p_i^2 q_1^2 \times \ldots \times \sum p_i^n q_1^n}
\] (6.8)

\[
F_{cv}^{ch} = \left( L_{cv} \times P_{cv} \right)^{1/2}
\] (6.9)
From above equation, it can be seen that the value of a chain Laspeyres volume index for year \( y \) is equal to the chain Laspeyres volume index for year \( y-1 \) multiplied by the Laspeyres volume index from year \( y-1 \) to year \( y \):

\[
L_{CV}^{y} = \frac{\sum_{i=1}^{n} P_{i}^{y-1} Q_{i}}{\sum_{i=1}^{n} P_{i}^{y-1} Q_{i}^{y-1}} \times L_{CV}^{y-1} \to y
\]

The Laspeyres volume index from year \( y-1 \) to year \( y \) is simply the quotient of the value at year \( y \) in the average prices of year \( y-1 \) and the current price value in year \( y-1 \). Similar expressions can be derived for the Paasche and Fisher volume indices.

### Chain Index Example

<table>
<thead>
<tr>
<th>Chain-linked volume indices</th>
<th>Sales of beef and chicken</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{CV}^{1} )</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>( L_{CV}^{2} )</td>
<td>100.00 x 1.050</td>
<td>105.00</td>
</tr>
<tr>
<td>( L_{CV}^{3} )</td>
<td>105.00 x 1.041</td>
<td>109.32</td>
</tr>
<tr>
<td>( L_{CV}^{4} )</td>
<td>109.32 x 1.154</td>
<td>126.18</td>
</tr>
</tbody>
</table>

### Direct volume indices

\[
L_{DV}^{1} = \frac{17 \times 100 + 17 \times 200}{40.00} = 127.50
\]
\[
P_{DV}^{1} = \frac{38.65}{20 \times 1.30 + 10 \times 2.15} = 123.47
\]

Also refer to reference material in your folder
**Annual Chain Indices in Annual National Accounts-Recommendations**

* 2008 SNA recommends ANA should be balanced in both current prices and in volume terms using supply and use tables
* Volume estimates are best derived in the average prices of the previous year rather than some distant base year
  - Assumptions of fixed relationships in volume terms are usually more likely to hold in the previous year’s average prices than in the prices of some distant base year;
  - Growth rates of volumes and prices are less affected by compositional change.

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**Annual Chain Indices in Annual National Accounts-Recommendations**

* Compute annual Fisher volume indices from data balanced data in supply and use tables (SUTs),
* Derive both the constituent Laspeyres and Paasche volume indices from data balanced in SUTs
* Derive the numerator of the Laspeyres volume index from year \((y-1)\) to year \(y\) by balancing the SUT for the current year \((y)\) in the average prices of the previous year \((y-1)\)
* Derive the denominator of the Laspeyres volume index by balancing the SUT for the previous year \((y-1)\) in the average prices of the previous year \((y-1)\).

* To derive the numerator for the Paasche volume index from year \(y-1\) to year \(y\) requires balancing the SUT for the current year \((y)\) in the average prices of the current year \((y)\) and to derive the denominator requires balancing the SUT for the previous year \((y-1)\) in the average prices of the current year \((y)\).

This last SUT (values for year \(y-1\) in the average prices of year \(y\)) is not required for compiling annual chain Laspeyres indices, and so the compilation of annual chain Fisher indices from balanced SUTs is somewhat more demanding.
Derivation of quarterly chain-linked volume indices from quarterly data with quarterly base periods is no different to compiling annual chain-linked volume indices from annual data with annual base periods;

2008 SNA recommends seasonally adjusted indices

If quarterly volume indices are to have quarterly base periods and be linked each quarter, then it should only be done using seasonally adjusted data (2008 SNA recommendation)

Considerations:
* Volatile prices/volumes in seasonally adjusted indices
* Do not use quarterly chains for quarterly base periods
* Alternatively use annualized quarterly indices