KOF

Concepts and Methods and Examples of Composite Business Cycle Indicators

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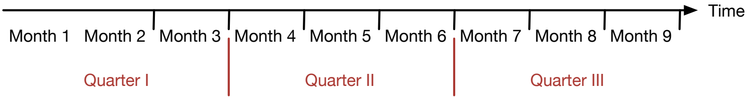
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Outline I

Outline

????casting


Time
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Quarter I
Quarter II
Quarter III

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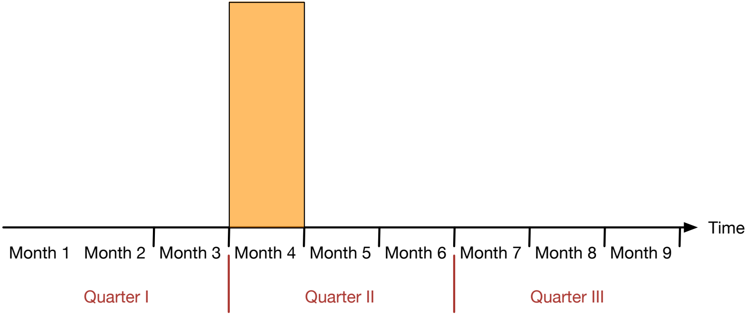
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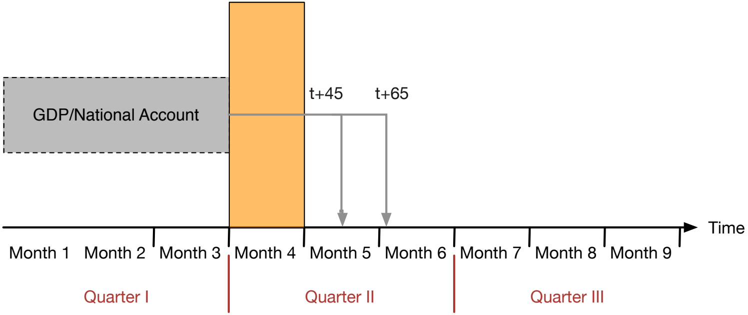

GDP/National Account
t+45
t+65
Time
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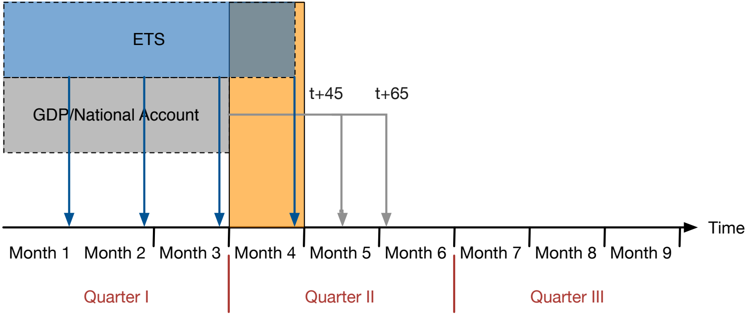
????casting


ETS
GDP/National Account
t+45
t+65
Time
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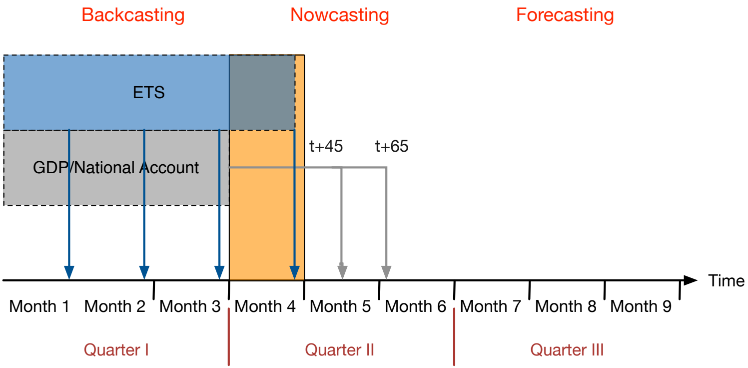
Backcasting
Nowcasting
Forecasting
ETS
GDP/National Account
t+45
t+65
Time
Month 1
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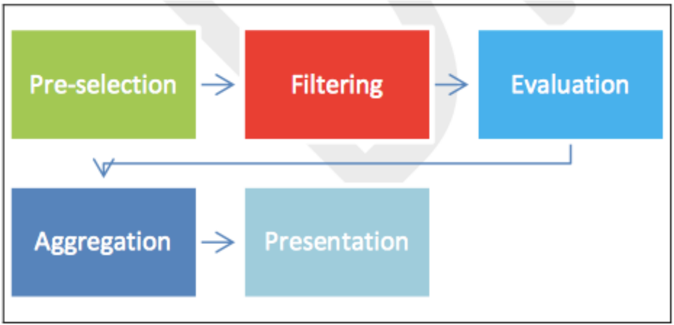
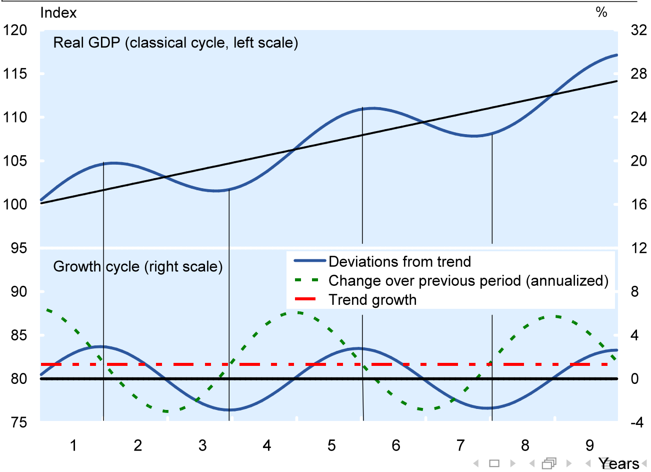
Production process of composite indicators



Outline

What is a business cycle?

Classical cycle (example: four year sine-wave fluctation of real GDP around a linear trend) and growth cycle



Business Cycles

Mitchell 1927:

The phenomena with which business cycles may be confused are (1) changes in business conditions which occur between the dates of "crises," (2) fluctuations which affect a minor portion of the economic activities of a business community, (3) fluctuations which recur every year, and (4) the less definitely established secondary trends and "long waves." From the first of these related species, business cycles are distinguished by the fact that each cycle includes one wave of rising and falling, or falling and rising activity, whereas the intervals between "crises" often include two and some times include three such waves. From the second species, business cycles are distinguished by their wider inclusiveness. From the third species they are distinguished by not recurring annually. From the fourth species they are distinguished by their briefer time-span.

Business Cycles

Mitchell 1927:

Following the lines of this analysis, we indicate both the generic features and the distinguishing characteristics of business cycles by saying that they are recurrences of rise and decline in activity, affecting most of the economic processes of communities with well-developed business organization, not divisible into waves of amplitudes nearly equal to their own, and averaging in communities at different stages of economic development from about three to about six or seven years in duration.

Reference Series

To assess the quality of business cycle indicators **reference series** are needed.

Usually Gross Domestic Product (GDP) is used as reference. In the absence GDP of synthetic activity measures or indicators of key parts of the economy (e.g. industrial production) could be considered.

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The reference series should

- be reliable
- contain a broad/important range of economic activity
- **be in a quarterly or monthly frequency**

Example: Reference Series for the Global Economy

- **Global GDP of the IMF (yearly), IMF World Economic Outlook Databases**

Example: Reference Series for the Global Economy

- Global GDP of the IMF (yearly), IMF World Economic Outlook Databases
- **GDP of OECD (quarterly), OECD Database**

Example: Reference Series for the Global Economy

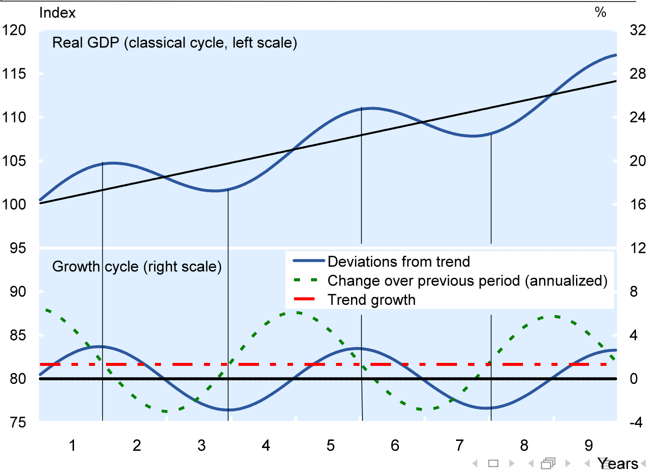
- Global GDP of the IMF (yearly), IMF World Economic Outlook Databases
- GDP of OECD (quarterly), OECD Database
- **Industrial Production of OECD (monthly), OECD Database**

Reference Series

So, the construction of composite business cycle indicators relies on a reference series.
One can in general classify the construction of composite indicators whether they us a reference series or not. Examples of composite indicators which do not rely on a reference series are quality of life or competitiveness of an economy.

What is a business cycle?

Classical cycle **(example: four year sine-wave fluctation of real GDP around a linear trend) and growth cycle**



Types of Cycles

Different Cycles

- **Classic Cycle**

Types of Cycles

Different Cycles

- Classic Cycle
- **Growth Cycle**

Types of Cycles

Different Cycles

- Classic Cycle
- Growth Cycle
- **Growth Rate Cycle**

Why is there a trend development?

What drives growth?

- **Population**

Why is there a trend development?

What drives growth?

- Population
- **Capital accumulation**

Why is there a trend development?

What drives growth?

- Population
- Capital accumulation
- **Technical progress**

Time Series Composition

Additive Model

Additive components model:

$$y_t = m_t + k_t + s_t + \epsilon_t, \quad t = 1, ..., n,$$

with

m_t trend component

k_t cyclical component (business cycle)

s_t seasonal component

ϵ_t irregular component

Time Series Composition

Additive Model

- Sometimes trend an business cycle are put together to a smooth component g_t. Then the model is

$$y_t = g_t + s_t + \epsilon_t, \quad t = 1, ..., n.$$

- The additive model could in principal enlarged, adding additional effects of regressors x_t. This leads to

$$y_t = g_t + s_t + x_t\beta + \epsilon_t, \quad t = 1, ..., n.$$

- In this way for example calendar effects or political measures could be captured.

Time Series Composition

Multiplicative Model

Multiplicative components model:

$$y_t = g_t \cdot s_t \cdot \epsilon_t, \quad t = 1, ..., n.$$

Trend estimation with Filters

A conventional definition of business cycle emphasises fluctuations of between about 1.5 years and 8 years. Longer fluctuations are regarded as trend. Shorter fluctuations contain short term fluctuations, wether effects, random effect, measurement errors etc.

One way to extract the smooth component and the business cycle is the application of filters like the Hodrick-Prescott filter and the Baxter-King filter.

Trend Estimation with Filters

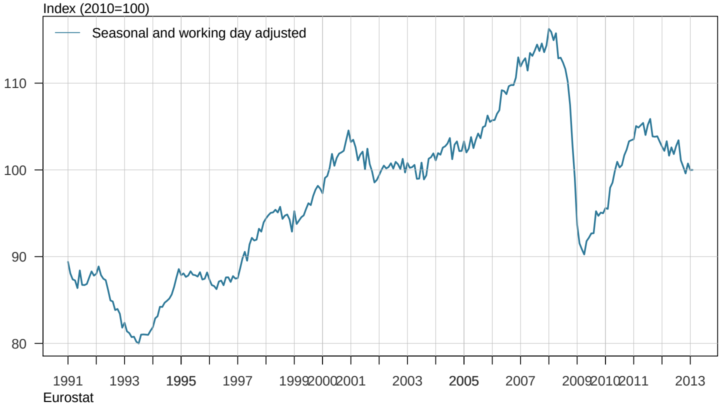
Hodrick-Prescott Filter (H-P Filter)

Estimations result from minimizing:

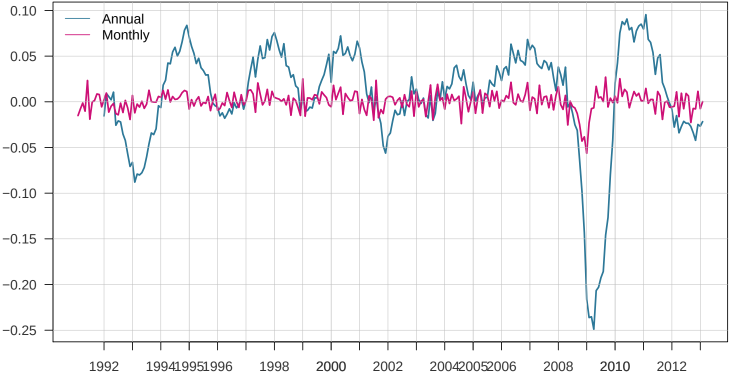
$$\sum_{t=1}^{T}(y_t - \mu_t)^2 + \lambda[(\mu_{t+1} - \mu_t) - (\mu_t - \mu_{t-1})]^2$$

and is a solution to the problem of minimizing the deviations between y and μ subject to a condition on the smoothness of the estimated component.

Example: Euro Area Manufacturing Production



Example: Growth of Euro Area Manufacturing Production



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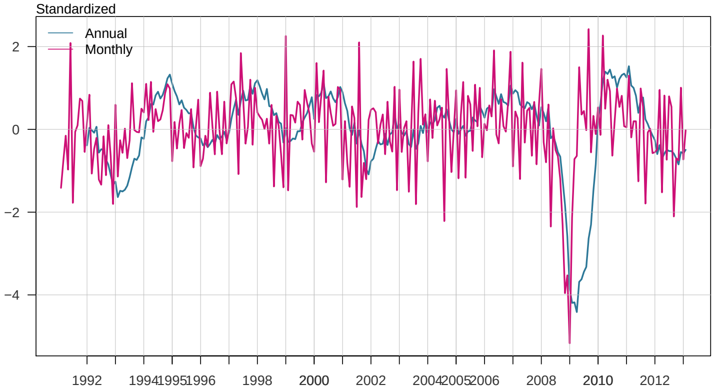

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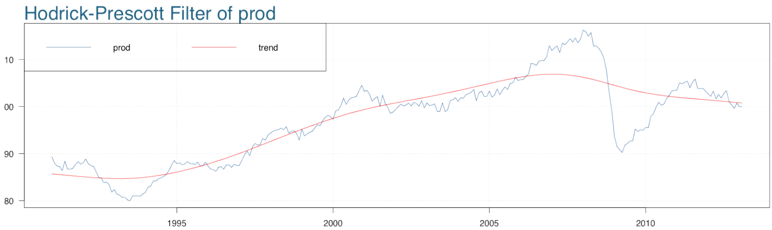

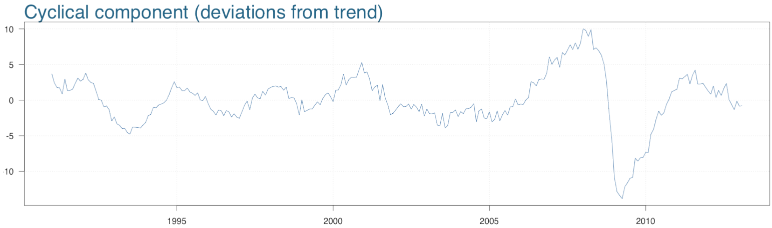

Outline

Searching for Indicators

Characteristics of good indicators:

- **meaningful und reliable**

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- **timely available**

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- **stable relationship with the reference series**

Searching for Indicators

Characteristics of good indicators:

- meaningful und reliable
- timely available
- after publication no big revisions
- leading or coincident for the business cycle, so that timely signals are given
- stable relationship with the reference series
- **clear signal with minor noise**

Types of Indicators

Indicators can be divided into

- **leading indicators**

Types of Indicators

Indicators can be divided into

- leading indicators
- **coincident indicators**

Types of Indicators

Indicators can be divided into

- leading indicators
- coincident indicators
- **lagging indicators**

Potential indicators

Potential leading indicators are classified to one of four types of economic rationale, shown below, that can be used to assess their suitability as leading indicators.

- ***Early stage:* indicators measuring early stages of production, such as new orders, order books, construction approvals, etc.**

Potential indicators

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- *Early stage:* indicators measuring early stages of production, such as new orders, order books, construction approvals, etc.
- ***Rapidly responsive:* indicators responding rapidly to changes in economic activity such as average hours worked, profits and stocks.**

Potential indicators

- ***Expectation-sensitive:*** **indicators measuring, or sensitive to, expectations, such as stock prices, raw material prices and expectations based on business survey data concerning production or the general economic situation/climate e.g. confidence indicators.**

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- ***Prime movers:*** **indicators relating to monetary policy and foreign economic developments such as money supply, terms of trade, etc.**

Search for possible Indicators

The list of possible indicators should contain the following information:

Indicator	Source	Notes	Meaning	Frequency	Publication lag	Revisions	Date first publication

In column "Meaning" one should identify why this indicator could be important for the economy (e.g. is an important sector with high value added)

Later this table is expanded with results from statistical analyses.

Composite Indicators

The advantage of composite indicators over the individual component series is that they achieve a **better trade-off between responsiveness and stability.** Composite indicators can be constructed to have **fewer false alarms and fewer missed turning points** than its individual components; moreover they tend to have **more stable lead-times.** Finally, the composites have the capacity to react to various sources of economic fluctuations and at the same time can be resilient to perturbations affecting only one of the components.

What others do

OECD:

To get some ideas about possible indicators look for example at the OECD website.

What others do: OECD

Look at the different country indicators. There are various types of indicators. E.g.:

- Production, stock of orders, employment, unfilled job vacancies, new car registrations, housing starts, nights spend in hotels
- business tendency surveys
- consumer surveys
- various price figures and share prices, terms of trade, exchange rate, silver price
- interest rates (spreads), bank credits
- indicators of other countries

.... and much more. (OECD List)

History of OECD Composite Leading Indicators

The OECD system of composite leading indicators was first developed in the early 1970s amidst renewed interest in business cycle research - a direct consequence of the 1969-1970 recession in developed economies. The deeper and more global recession that followed in the mid-70s reinforced the need for such a tool, leading to the creation of a dedicated OECD Working Party on Cyclical Analysis and Leading Indicators in 1978.

Purpose of the OECD Composite Leading Indicators

The objective of the OECD Composite Leading Indicators is to provide ”qualitative indicators of the business cycle outlook for the short term future”.

So what means

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- **qualitative?**

Purpose of the OECD Composite Leading Indicators

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So what means

- business cycle?
- qualitative?
- **short term?**

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- **Business cycle: Deviation from trend in GDP (since 2012, before deviation from trend in industrial production).**

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- **Qualitative: By design the indicators are primarily aimed at identifying turning-points but also tries to identify phases in the cycle and, albeit to a lesser extent, the acceleration/deceleration of the business cycle. The qualitative focus means that the indicators are not optimized for precise numeric forecasting. There is also a risk that one would intuitively interpret higher peaks and lower troughs as stronger/weaker growth. However, such conclusions may be misplaced, because the indicators are not optimized in this way.**

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- **Short term: The indicators are designed to have a typical lead of between 6 and 9 months. However, in practice the timeliness of data releases affects information lead times.**

Building Blocks of OECD Composite Leading Indicators

- From the candidate component series factors like seasonal pattern, outliers, trend and noise (applying the HP-filter) are removed.
- Candidate series are standardized.
- Assessment of components (turning point analysis with Bry-Boschan procedure, cross-correlations).
- Calculation of composite indicator (equal weighting of components) and assessment (turning points, cross-correlations).

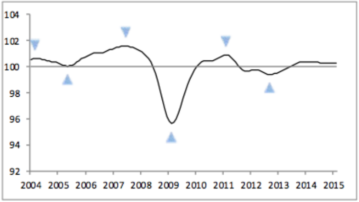
Presentation of OECD Composite Leading Indicators

The raw composite indicator is the average of the de-trended, smoothed and normalized component series.

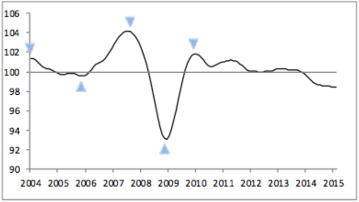
- Amplitude adjusted: The amplitude adjusted indicator rescales the raw indicator to match the amplitudes of the business cycle (i.e. the de-trended and smoothed but not normalized/standardized reference series).
- Trend restored: It is the product of the trend of the reference series and the amplitude adjusted composite indicator. This transformation facilitates analyses of the classical business cycle.
- Annual growth rate: The annual growth rates, calculated from the rend restored composite indicator. Some analysts prefer this type of indicator because they are used to interpret annual changes (of the reference series).

What others do: OECD

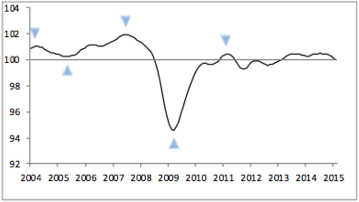
Stable growth momentum in the OECD area



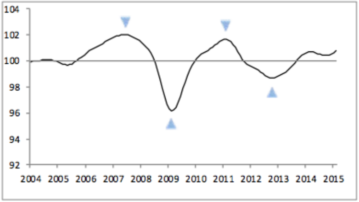
Easing growth in China



Stable growth momentum in the United States



Positive change in momentum in the Euro area



The Conference Board

- This indicator approach originated in the mid-1930s at the National Bureau of Economic Research (NBER) with the work of Wesley Mitchell and Arthur Burns
- Starting in the late 1960s, the U.S. Department of Commerce began publishing the composite indexes
- In late 1995, the indicator program was privatized and The Conference Board took over

The Conference Board

The Conference Board leading indicator for the U.S. uses the following data:

- Average weekly hours, manufacturing
- Average weekly initial claims for unemployment insurance
- Manufacturers' new orders, consumer goods and materials
- ISM new orders index
- Manufacturers' new orders, non-defense capital goods excl. aircraft
- Building permits, new private housing units Stock prices, 500 common stocks

The Conference Board

- Leading Credit Index
- Interest rate spread, 10 year Treasury bonds less federal funds
- Avg. consumer expectations for business conditions

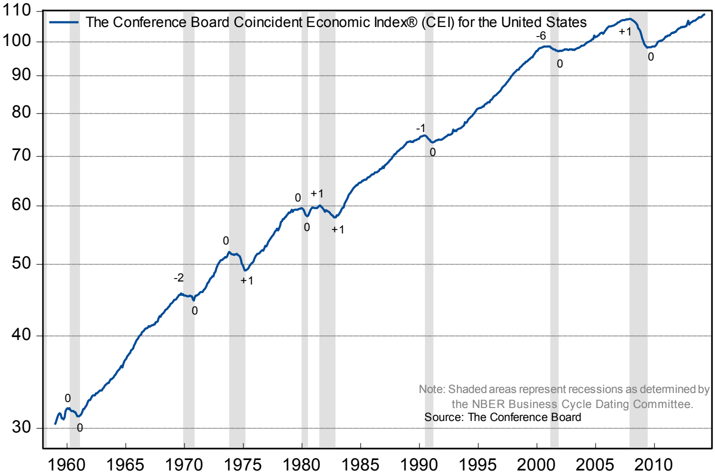
Standard deviations of monthly changes of the variables are used to calculate a weighted average of the variables.

The Conference Board

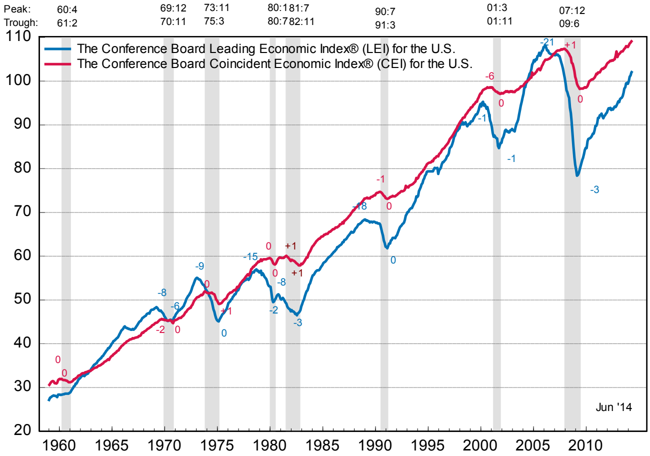
The Conference Board coincident indicator for the U.S. uses the following data:

- Employees on nonagricultural payrolls
- Personal income less transfer payments
- Industrial production
- Manufacturing and trade sales

The Conference Board U.S. Coincident Indicator



The Conference Board U.S. Leading Indicator



Note: Shaded areas represent recessions.
Source: The Conference Board

The Conference Board

Construction of Conference Board composite indicators. The components (variables) are

- Seasonal adjusted
- Deflated
- Volatility adjusted
- Aggregated
- In some cases trend adjusted (the leading indicator is adjusted to the trend of the coincident indicator)
- An index is calculated

The Conference Board

Construction of Composite Index

1. Calculate month to month changes for each component. For components which are in percent form, simple arithmetic differences are calculated: $r_{i,t} = X_{i,t} - X_{i,t-1}$. In all other cases a **symmetric percentage change** formula is used: $r_{i,t} = 200 \cdot \frac{X_{i,t} - X_{i,t-1}}{X_{i,t} + X_{i,t-1}}$.
2. Adjust the month-to-month changes by multiplying them by the component's standardization factor, w_i. This results in $c_{i,t} = w_i \cdot r_{i,t}$.

The Conference Board

3. Sum up components (variables) to calculate the monthly change in the composite indicator.
4. In the leading indicator demean the monthly change of the indicator obtained in 3.
5. In the leading indicator add the mean of the monthly changes in the coincident indicator as estimated trend component.

The Conference Board

6. Cumulate the (trend adjusted) monthly changes to the preliminary level of the index. The index is calculated recursively, starting from an initial value of 100 for the first month of the sample. Let $I_1 = 100$ denote the initial value of the index for the first month. If s_2 is the result from Step 5. in the second month, the preliminary index value is $I_2 = I_1 \cdot \frac{200+s_2}{200-s_2}$. Then the next month's preliminary index value is: $I_3 = I_2 \cdot \frac{200+s_3}{200-s_3}$, and so on for each month data that are available.
7. Rebase the index to an average 100 in the chosen base year. The preliminary index levels obtained in Step 6 are multiplied by 100, and divided by the mean of the preliminary levels of the index in the base year.

The Conference Board

The Conference Board Coincident Economic Index (CEI) for China: Components

- Value-Added Industrial Production (Billions of 2004 Yuan, deflated by PPI, S.A.)
- Retail Sales of Consumer Goods (Billions of 2004 Yuan, deflated by RPI, S.A.)
- Volume of Passenger Traffic (Person Bn-Kilo, S.A.)
- Electricity Production (Billions of KWH, S.A.)
- Manufacturing Employment (Person Mn, S.A.)

The Conference Board

The Conference Board Leading Economic Index (LEI) for China: Components

- Consumer Expectations Index
- Total Loans Issued by Financial Institutions (Billions of 2004 Yuan, deflated by PPI, S.A.)
- 5000 Industry Enterprises Diffusion Index: Raw Materials Supply (S.A.)
- PMI: Manufacturing: Supplier Delivery (S.A.)
- PMI: Manufacturing: New Export Orders (S.A.)
- Floor Space Started: Total (Thousands of Sq M, S.A.)

KOF Economic Barometer

Indicator for the Swiss business cycle. Relies strongly on economic tendency survey results, but not entirely. Uses Swiss indicators and foreign indicators.

KOF Economic Barometer: History

- **1976 Version**

KOF Economic Barometer: History

- 1976 Version
  - **Reference series: de-trended real GDP**

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  - **Number of variables selected: 6 (construction, manufacturing (2x), labour, money, stocks)**

KOF Economic Barometer: History

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- **1998 Version**

KOF Economic Barometer: History

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  - **Reference series: real y-o-y growth in GDP**

KOF Economic Barometer: History

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- 1998 Version
  - Reference series: real y-o-y growth in GDP
  - Number of variables selected: 6 (all from Business Tendency and Consumer surveys)
  - **Variables were low-pass filtered and then the first principal component was extracted**

KOF Economic Barometer: History

- **2006 Version**

KOF Economic Barometer: History

- 2006 Version
  - **Reference series: real y-o-y growth in financial, construction and core GDP (3 modules)**

KOF Economic Barometer: History

- 2006 Version
  - Reference series: real y-o-y growth in financial, construction and core GDP (3 modules)
  - **Number of variables selected: 25**

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- 2006 Version
  - Reference series: real y-o-y growth in financial, construction and core GDP (3 modules)
  - Number of variables selected: 25
  - **For each module the first principle component was extracted**

KOF Economic Barometer: History

- 2006 Version
  - Reference series: real y-o-y growth in financial, construction and core GDP (3 modules)
  - Number of variables selected: 25
  - For each module the first principle component was extracted
  - **Aggregate is filtered using end-point stable Direct Filter Approach (DFA) of Wildi (2008)**

KOF Economic Barometer: Construction of the 2004 Version

- **Objectives**

KOF Economic Barometer: Construction of the 2004 Version

- Objectives
  - **No longer use a filter for smoothing**

KOF Economic Barometer: Construction of the 2004 Version

- Objectives
  - No longer use a filter for smoothing
  - **Broaden the set of underlying time series**

KOF Economic Barometer: Construction of the 2004 Version

- Objectives
  - No longer use a filter for smoothing
  - Broaden the set of underlying time series
  - **Define a standardized procedure to select variables (Automatize and regularly apply the variable selection procedure)**

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  - Define a standardized procedure to select variables (Automatize and regularly apply the variable selection procedure)
- **Two production stages**

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- Two production stages
  - **Variable selection procedure**

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- Objectives
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  - Broaden the set of underlying time series
  - Define a standardized procedure to select variables (Automatize and regularly apply the variable selection procedure)
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  - Variable selection procedure
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KOF Economic Barometer: Construction of the 2004 Version

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  - **Construction of the leading indicator (extract the first principle component from the selected variables)**

KOF Economic Barometer: Reference Series

- **The KOF Barometer is an indicator published monthly**

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  - High frequency current growth rate are highly volatile, reflecting measurement errors, weather effects, working day effects, and alike
  - **The aim of the KOF Barometer is to signal the underlying business cycle - not high frequency fluctuations**

KOF Economic Barometer: Candidate Variables

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  - **BFS, SECO, OZD, SNB (24)**

KOF Economic Barometer: Candidate Variables

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  - theoretically expected sign of the correlation with the reference series
- **Except for year-over-year differences, X12-ARIMA is used to seasonally adjust all variables and their transformations.**

KOF Economic Barometer: Automated selection procedure

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- The computed cross-correlation surpasses a defined threshold.
- **Of those transformations that survive, we take the one that optimizes:** $\max U = |r^{\max}| \cdot \sqrt{h^{\max} + 1}$

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- **Finally, the variance of these variables is collapsed into a composite indicator as the first principal component.**

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- Finally, the variance of these variables is collapsed into a composite indicator as the first principal component.
- **This first principal component is standardised to have a mean of 100 and standard deviation of 10 during the observation window.**

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- Except for year-over-year differences, the seasonal factors are subtracted from all variables and their transformations. The seasonal factors are kept constant until the next vintage is constructed.
- We standardise the variables entering the KOF Barometer using their means and standard deviations estimated for the 10-year reference window.
- **The first principal component is constructed by multiplying the standardised variables with the loading coefficients derived for the reference period.**

KOF Economic Barometer: Out of sample production

- **We scale the constructed first principal component by the value of the standard deviation of the first principal component computed using the reference window.**

KOF Economic Barometer: Out of sample production

- We scale the constructed first principal component by the value of the standard deviation of the first principal component computed using the reference window.
- **We construct the KOF Barometer values by multiplying the standardised principal component by 10 and adding 100.**

KOF Economic Barometer: Yearly updates in September

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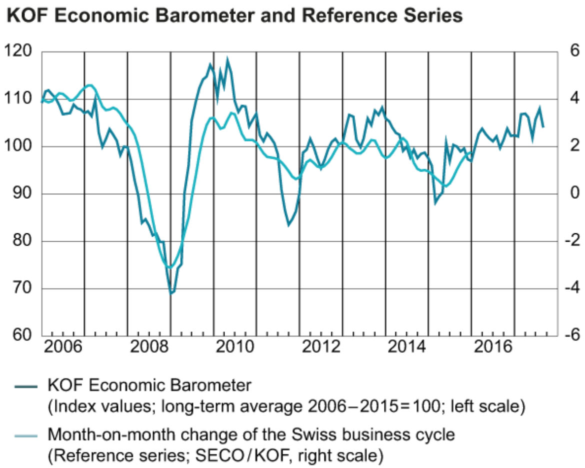
  - This vintage contains the first release of previous yearÃŢs growth by the SFSO
- **The subsequent quarterly release of SECO incorporates this annual information**

KOF Economic Barometer



What next?

When we have collected a list of indicators, we have to look for each series whether we need a

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When we have collected a list of indicators, we have to look for each series whether we need a

- seasonal adjustment
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Outline

Seasonal Adjustment

Model

- Additive model $y_t = g_t + s_t + \epsilon_t, \quad t = 1, ..., n$
- Multiplicative model $y_t = g_t \cdot s_t \cdot \epsilon_t, \quad t = 1, ..., n$

Seasonal Adjustment

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Adjustment

- Additive model

$$y_t - s_t = g_t + \epsilon_t, \quad t = 1, ..., n$$

- Multiplicative model

$$\frac{y_t}{s_t} = g_t \cdot \epsilon_t, \quad t = 1, ..., n$$

Seasonal Adjustment

Steps

To seasonally adjust we have to:

- **Estimate the smooth component**

Seasonal Adjustment

Steps

To seasonally adjust we have to:

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- **Adjust series for the smooth component**

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To seasonally adjust we have to:

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- Adjust series for the smooth component
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Seasonal Adjustment

Steps

To seasonally adjust we have to:

- Estimate the smooth component
- Adjust series for the smooth component
- Estimate seasonal factors
- **Adjust original series for seasonal factors**

A simple procedure for illustration

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Census X13ARIMA

Census X13ARIMA is a sophisticated filter based method for seasonal adjustment.

Census X13ARIMA

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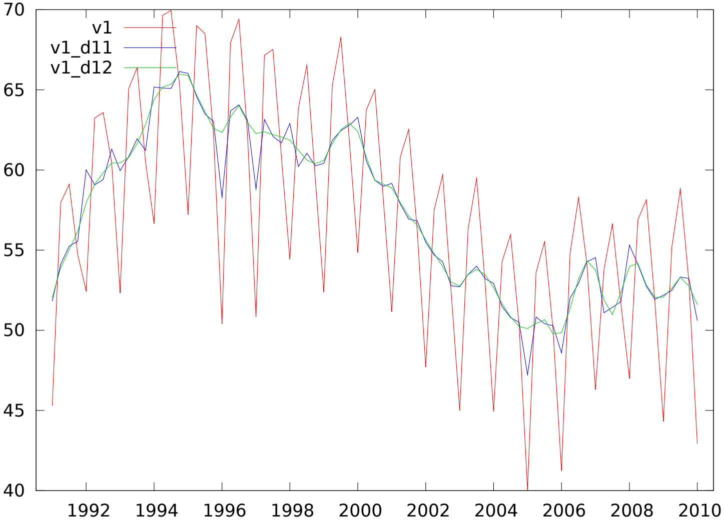
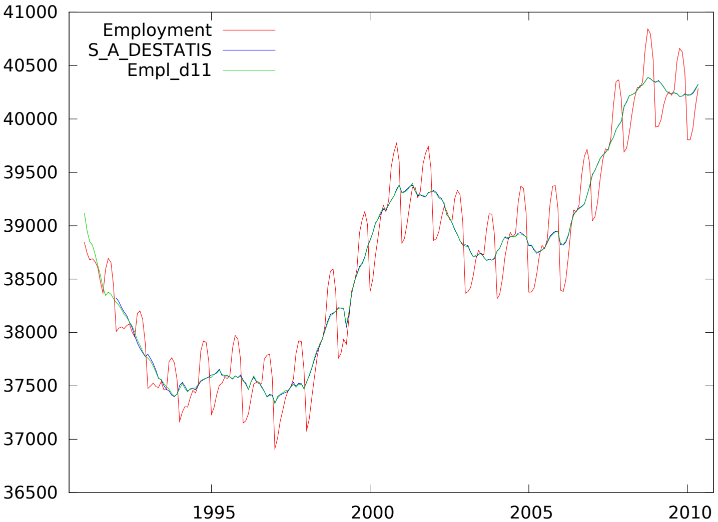
Maintained by U.S. Census Bureau (Census Homepage)

Example Construction



Example Employment



Outline

Autocorrelation and Cross-correlation

The common Bravais-Pearson correlation coefficient for two variables Y and X is

$$
\begin{aligned}
r_{xy} &= \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} \\
&= \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2)(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2)}}
\end{aligned}
$$

Autocorrelation and Cross-correlation

For a stationary time series $x_1, x_2, x_3, ..., x_n$ and $-n < h < n$ the autocorrelation function is estimated with

$$\hat{\gamma}_h = \frac{\sum_{t=1}^{n-|h|}(x_{t+|h|} - \bar{x})(x_t - \bar{x})}{\sqrt{\sum_{t=1}^{n}(x_t - \bar{x})^2 \sum_{t=1}^{n}(x_t - \bar{x})^2}}$$

Autocorrelation and Cross-correlation

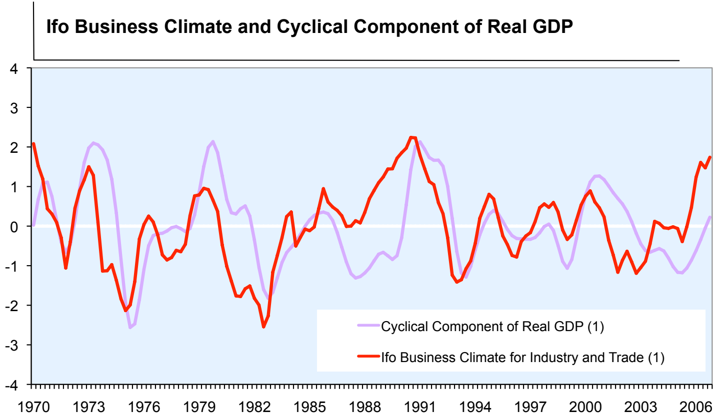
For two jointly stationary time series $x_1, x_2, x_3, ..., x_n$; $y_1, y_2, y_3, ..., y_n$ and $0 \leq h < n$ the cross-correlation function is estimated with

$$\hat{\rho}_{xy}(h) = \frac{\sum_{t=1+h}^{n}(x_t - \bar{x})(y_{t-h} - \bar{y})}{\sqrt{\sum_{t=1}^{n}(x_t - \bar{x})^2 \sum_{t=1}^{n}(y_t - \bar{y})^2}}$$

Resp. for $-n < h \leq 0$

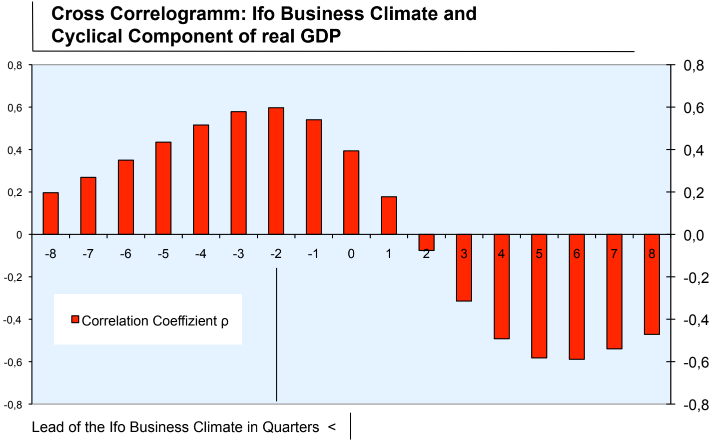
$$\hat{\rho}_{xy}(h) = \frac{\sum_{t=1}^{n-|h|}(x_t - \bar{x})(y_{t-h} - \bar{y})}{\sqrt{\sum_{t=1}^{n}(x_t - \bar{x})^2 \sum_{t=1}^{n}(y_t - \bar{y})^2}}$$

Example



1) Standardized.
Source: Federal Statistical Office, Ifo Business Survey.

Example



Source: Statistical Office, Ifo Business Survey.

Algorithm for dating turning points

To minimize subjective assessments and to have a fast routine for dating turning points we use an algorithm.

An grass-route work is:
G. Bry, C. Boschan (1971), ”Cyclical Analysis of Time Series: Selected Procedures and Computer Programs”, NBER Technical Paper no 20.

Another important work is:
D. Harding, A. Pagan (2002), ”Dissecting the Cycle: a Methodological Investigation”, Journal of Monetary Economics, no 49, pp. 365-381.

Algorithm for dating turning points

Harding, Pagan: Minimum needs for an algorithm

1. Determination of a potential set of turning points i.e. the peaks and troughs in a series.
2. A procedure for ensuring that peaks and troughs alternate.
3. A set of rules that re-combine the turning points established after steps one and two in order to satisfy pre-determined criteria concerning the duration and amplitudes of phases and complete cycles; what we will refer to as "censoring rules".

Algorithm for dating turning points

Monthly data (classical and growth cycles)

A local peak (trough) is occurring at time t whenever $\{y_t > (<) y_{tk}\}, k = 1, ..., K$, where K is generally set to five. A phase must last at least six months and a complete cycle should have a minimum duration of fifteen months.

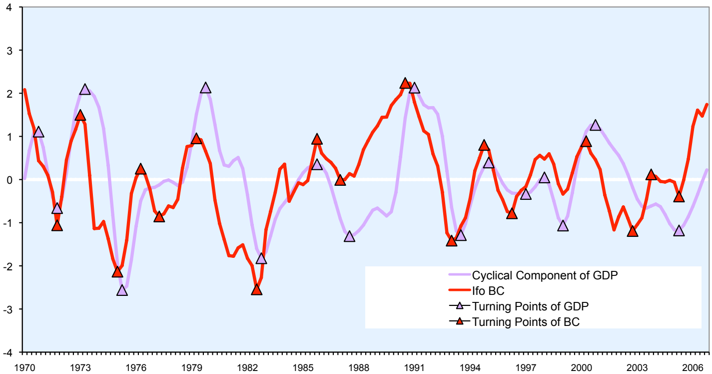
Quarterly data

Put $K = 2$ i.e. $\{\Delta_2 y_t > 0, \Delta y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}$, as this ensures that y_t is a local maximum relative to the two quarters (six months) on either side of y_t.

In addition (monthly) data are sometimes smoothed before dating. A moving average or a spencer curve is usually applied to reduce noise.

Example

Turning Points of the German Business Cycle and Ifo Business Climate

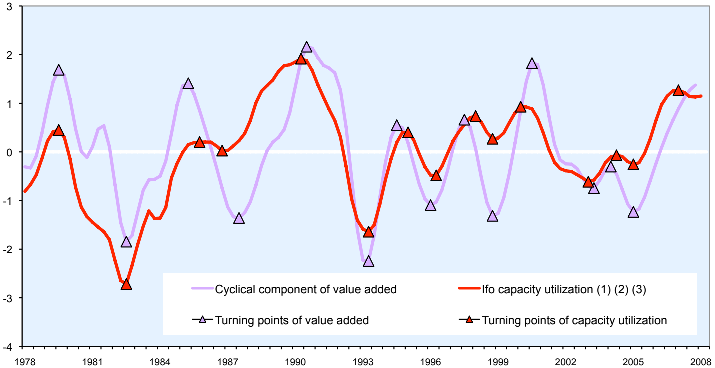


1) Standardized values

Source: Statistisches Bundesamt, ifo Konjunkturtest.

Example

Turning Points of Real Value Added in Manufacturing in Germany and Ifo Capacity Utilization



1) Standardized values.-2) Including food.-3) Smoothed with local weighted polynomial regression.

Source: DESTATIS, Ifo Business Cycle Test.

Outline

Approaches

Question:

How can we condense information contained in various indicators into one (or at least in a view) indicator(s)?

- Classical (NBER)
- Factor analysis and Principal Components

Steps in Classical Approach

- Choose and classify indicators (detrending, cross-correlations, turning points, co-spectral analysis)
- Standardize indicators
- Average indicators (and standardize)

Factor Analysis

Factor Models

- a common force drives the dynamics of all variables
- common force, also known as common factor, is typically of low dimension and is not directly observed because every macroeconomic variable embodies some idiosyncratic noise or short term movements
- factor models clean every variable from these idiosyncratic movements and estimate the common component in every series

Factor Analysis

Let Z_t be a N-dimensional multiple time series. It is assumed that (Z_t) admit a factor model representation with r common latent factors F_t,

$$Z_t = \Lambda F_t + e_t \qquad (1)$$

where e_t is a $N \times 1$ vector of idiosyncratic disturbances.

Three Approaches

- Model based: factor estimation with unobserved components models and the Kalman filter
- Nonparametric: dynamic factor analysis by Forni et al.
- Nonparametric: principal components

Model Based: Unobserved Components

The model consists of two stochastic components: the common unobserved variable, or index c_t and an n dimensional component, u_t, that represents idiosyncratic movements in the series and measurement error. The formulation of the model is:

$$\begin{aligned} z_t &= \beta + \gamma(L)c_t + u_t \,(\textit{measurement equation}) \\ \phi(L)c_t &= \delta + \nu_t \,(\textit{state equation}) \\ D(L)u_t &= \epsilon_t \end{aligned}$$

Estimation is done by the Kalman filter.

Literature: Stock and Watson (1988), A Probability Model of the Coincident Economic Indicators. Journal of Business and Economic Statistics, 147-162.

Dynamic Factor Model (Forni et al.)

The dynamic factor model is:

$$z_{it} = \lambda_i(L) f_t + e_{it} \tag{2}$$

with $\lambda_i(L)$ a lag polynomial.

Approximative Dynamic Model and Principal Components (Stock, Watson Approach)

Under some assumptions the dynamic factor model

$$z_{it} = \lambda_i(L)f_t + e_{it} \tag{3}$$

can be rewritten as

$$Z_t = \Lambda F_t + e_t \tag{4}$$

with $F_t = (f_t' f_{t-1}' ... f_{t-q}')$.

Estimation can be done by principal components.

Literature: Stock and Watson (2002), Macroeconomic Forecasting Using Diffusion Indexes. Journal of Business and Economic Statistics, 147-162.

Comparison of the Nonparametric Approaches

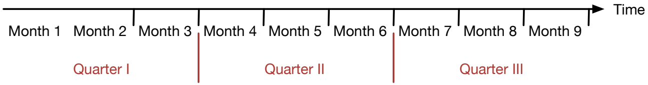
- The static model requires only the specification of r. The dynamic method requires input of four parameters.
- Estimation of static model is much more simple.
- A drawback of the static estimator is that it does not take into account the dynamics of the factors, if they exist.

Neither estimator necessarily dominates the other.

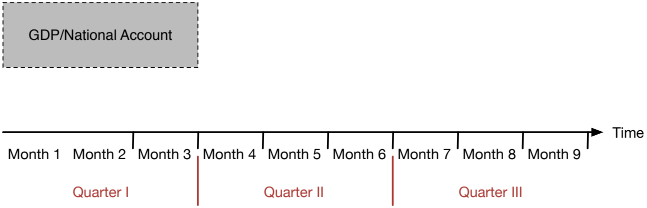
Literature: Boivin J., Ng S. (2005), Understanding and Comparing Factor-Based Forecasts. International Journal of Central Banking, 117-151.

Outline

????casting


Time
Month 1
Month 2
Month 3
Month 4
Month 5
Month 6
Month 7
Month 8
Month 9
Quarter I
Quarter II
Quarter III

????casting


GDP/National Account
Time
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Month 3
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Month 5
Month 6
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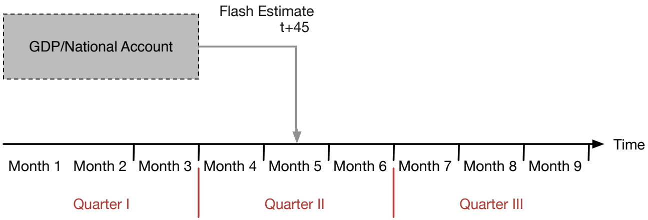
Quarter III

????casting


Flash Estimate
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GDP/National Account
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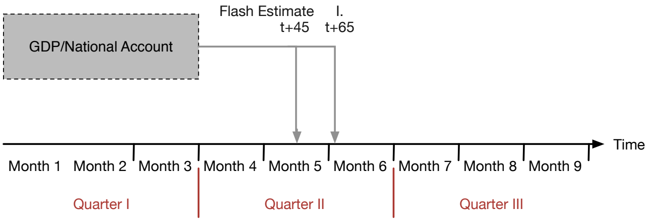
GDP/National Account
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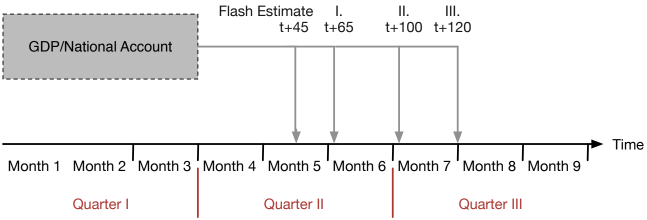
GDP/National Account
Flash Estimate
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t+65
II.
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III.
t+120
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Month 7
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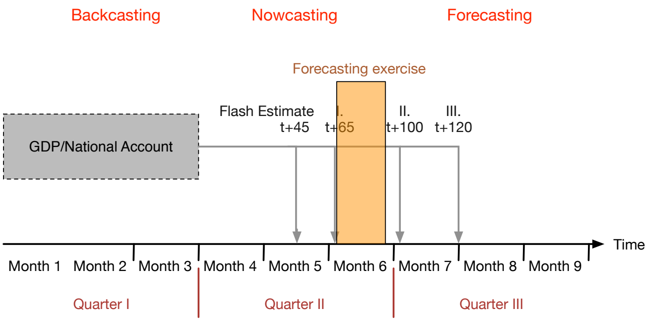
Backcasting
Nowcasting
Forecasting
Forecasting exercise
GDP/National Account
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t+65
II.
t+100
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t+120
Time
Month 1
Month 2
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Month 5
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Month 9
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Quarter II
Quarter III

Autoregressive Distributed Lag Model

With Y_t the target variable at time t and IN_t an (composite) indicator at time t one can try to estimate the model

$$Y_t = \alpha + \sum_{i=1}^{m} \beta_i Y_{t-i} + \sum_{j=0}^{n} \gamma_j IN_{t-j} + \epsilon_t,$$

with error term ϵ.
This approach can be used to test whether the coefficients γ are significant different from zero. In addition this model can be used to calculate indicator based forecasts.

Error Correction Model

With Y_t the target variable at time t and X_t an independent variable. An Error Correction Model is

$$\Delta_1 Y_t = \alpha + \zeta Z_{t-1} + \sum_{i=1}^{m} \beta_i \Delta_1 Y_{t-i} + \sum_{j=0}^{n} \gamma_j \Delta_1 X_{t-j} + \epsilon_t,$$

with error term ϵ and

$$Z_t = Y_t - bX_t.$$

$\Delta_1 Y$ will respond negatively to Z since Z is the deviation from the long run equilibrium state ($Z = 0$). ζ gives the speed of adjustment to the equilibrium.

Error Correction Model

- Estimates short term and long term effects
- Applications to stationary and nonstationary data
- Estimation with OLS

Error Correction Model

- Motivating ECM with cointegrated data
  - Handles nonstationary cointegrated variables
  - Two step estimation procedure
  - Models with only differenced variables may ignore long term efects
- Motivating ECM with stationary data
  - Long and short term effects
  - ECM and ADL are equivalent

Mixed Frequencies

Often time series with different frequency are used for forecasting. So one may want forecast quarterly GDP with a monthly composite indicator. The objective is to forecast a

lower-frequency variable, Y, sampled at periods denoted by time index t. Past realizations of the lower-frequency variable are denoted by the lag operator, L. For example, if Y_t is the quarterly GDP, then the GDP one quarter prior would be the first lag of Y_t, $LY_t = Y_{t-1}$, two months prior would be $L^2 Y_t = Y_{t-2}$, and so on. In addition to lags of Y, we are

interested in the information content of a higher-frequency variable, X, sampled m times between samples of Y (e.g., between $t-1$ and t). L_{HF} denotes the lag operator for the higher-frequency variable. If X_t is the monthly indicator, then $L_{HF}X_t$ denotes the indicator value of the last month of the

Time Aggregation

One solution to the problem of mixed sampling frequencies is to convert higher-frequency data to match the sampling rate of the lower-frequency data. One can for example calculate averages of X between samples of the low frequency variable:

$$\bar{X}_t = \frac{1}{m} \sum_{k=1}^{m} L_{LH}^{k} X_t .$$

With the two variables in the same time domain on can use a regression model for forecasting:

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{j=1}^{n} \gamma_j L^j \bar{X}_t + \epsilon_t$$

Time Aggregation

The standard aggregation methods depend on the stock/flow nature of the variables and, typically, it is the average of the high-frequency variables over one low-frequency period for stocks, and the sum for flows.

Taking the latest available value of the higher frequency variable is another option for both stock and flow variables. The underlying assumption is that the information of the previous high-frequency periods is reflected in the latest value, representative of the whole low-frequency period.

Bridge Models

With time aggregation often the problem occurs, that at a specific time point not all of the high-frequency data is already available. The Bridge Model Approach consist of two steps:

1. Forecasting the high-frequency series with time series methods (e.g. ARMA) for all time points which are needed for time aggregation.
2. An ADL model is used at the low frequency to obtain forecasts for Y.

Step Functions

The time aggregation method assumes the slope coefficients on each of the individual observations of X are equal. Alternatively, one could assume that each of the slope coefficients for each k sampling of X are unique. This model, including one lag of the predictor X $(n = 1)$, is

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{k=1}^{m} \gamma_k L_{HF}^k X_t + \epsilon_t.$$

The model can be estimated with OLS.

Step Functions

Once the model is extended to multiple lags, the number of parameters could become quite large (especially, when the high frequency data is of higher frequency than monthly). A more general model is

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{k=1}^{m \cdot n} \gamma_k L_{HF}^k X_t + \epsilon_t$$

which allows for up to n lower-frequency lags.So the problem with this type of models is, that the number of coefficient can get huge.

MIDAS (Mixed Data Sampling)

The time-averaging model is parsimonious but discards any information about the timing of innovations to higher-frequency data. The step-weighting model preserves the timing information but requires the user to estimate a potentially large number of parameters. To solve the problem of parameter proliferation while preserving some timing information MIDAS models can be used:

$$Y_t = \alpha + \sum_{i=1}^{p} \beta_i L^i Y_t + \sum_{k=1}^{m} \Phi(k;\theta) L_{HF}^k X_t + \epsilon_t .$$

where the function $\Phi(k;\theta)$ is a polynomial that determines the weights for temporal aggregation. The weighting function, $\Phi(k;\theta)$, can have any number of functional forms; the desire here is to achieve flexibility while maintaining parsimony.

MIDAS (Mixed Data Sampling)

Suggestions for $\Phi(k;\theta)$ are often a beta function formulation or an exponential Almond specification. The later is

$$\Phi(k;\theta_1,\theta_2) = \frac{exp(\theta_1 k + \theta_2 k^2)}{\sum_{j=1}^{m} exp(\theta_1 j + \theta_2 j^2)}$$

In this case, simple time averaging is obtained when $\theta_1 = \theta_2 = 0$.

Outline

Turning Points in the Business Cycle

Generally, practitioners in business cycle analysis sometimes assume that economic cycles are constituted by an alternation of two conjonctural phases, namely a phase of high economic activity (or expansion) and a phase of low economic activity (or contraction). These phases can be defined in classical, growth or growth rate cycles. Sometimes also or than two phases are considered.

The objective of parametric models is to provide, at each date t, an estimated probability of being in a specific phase.

Binary response models

If there is a reference series and if the phases (dating) of the reference series are available a binary variable can defined that takes the value 1 when the economy belongs to one phase and 0 when it belongs to the other phase. This $0-1$ variable can be used for logit or probit regressions.

Logistic Regression

Let Y be a binary variable with values 0 and 1 and X a predictor (e.g. a composite indicator), the the logistic regression model (logit) is

$$Log\left[\frac{prob(Y_t = 1)}{1 - prob(Y_t = 1)}\right] = a + bx_t. \tag{5}$$

The model can be extended to contain lags of X and lags of Y.

Markov Switching

Markov switching models consist to the class of nonlinear time series models. They base on the idea of probability switching between various states (e.g. upswing and downswing). In the following Markov switching autoregressive models are discussed. Markov switching regression models use also explanatory variables.

Markov Switching

Hamilton (1989) considers the Markov switching autoregressive (MSA) model. Here the transition is driven by a two-state Markov chain. A time series x_t follows an MSA model if it satisfies:

$$x_t = \begin{cases} c_1 + \sum_{i=1}^{p} \phi_{1,i} x_{t-i} + a_{1,t} & \text{if } s_t = 1, \\ c_2 + \sum_{i=1}^{p} \phi_{2,i} x_{t-i} + a_{2,t} & \text{if } s_t = 2, \end{cases} \quad (6)$$

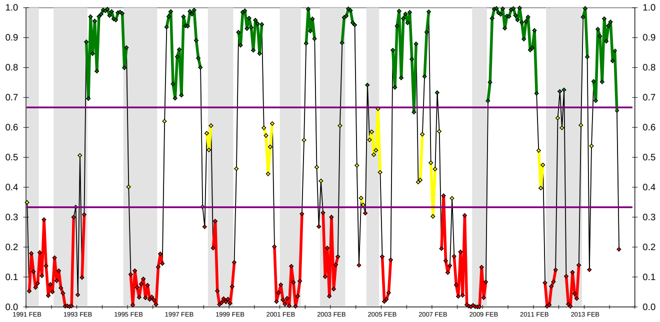
where s_t assumes values in $\{1, 2\}$ and is a first-order Markov chain with transition probabilities

$$P(s_t = 2 | s_{t-1} = 1) = w_1, \;\; P(s_t = 1 | s_{t-1} = 2) = w_2. \quad (7)$$

Markov Switching

The innovational series $\{a_{1,t}\}$ and $\{a_{2,t}\}$ are sequences of iid random variables with mean zero and finite variance and are independent of each other. A small w_i means that to model tends to stay longer in state i. In fact, $1/w_i$ is the expected duration of the process to stay in state i.

Example



Source: Calculations of the Ifo Institute.